$$\frac{1}{0!}+\frac{1}{3!}+\frac{1}{6!}+\cdots=\frac{e}{3}+\frac{2}{3\sqrt{e}}\cos\frac{\sqrt{3}}{2}$$

$$\frac{1}{1!}+\frac{1}{4!}+\frac{1}{7!}+\cdots=\frac{e}{3}-\frac{1}{3\sqrt{e}}\left(\cos\frac{\sqrt{3}}{2}-\sqrt{3}\sin\frac{\sqrt{3}}{2}\right)$$

$$\frac{1}{2!}+\frac{1}{5!}+\frac{1}{8!}+\cdots=\frac{e}{3}-\frac{1}{3\sqrt{e}}\left(\cos\frac{\sqrt{3}}{2}+\sqrt{3}\sin\frac{\sqrt{3}}{2}\right)$$

$$\frac{1}{0!}-\frac{1}{3!}+\frac{1}{6!}-\cdots=\frac{1}{3e}+\frac{2\sqrt{e}}{3}\cos\frac{\sqrt{3}}{2}$$

$$\frac{1}{1!}-\frac{1}{4!}+\frac{1}{7!}-\cdots=-\frac{1}{3e}+\frac{\sqrt{e}}{3}\left(\cos\frac{\sqrt{3}}{2}+\sqrt{3}\sin\frac{\sqrt{3}}{2}\right)$$

$$\frac{1}{2!}-\frac{1}{5!}+\frac{1}{8!}-\cdots=\frac{1}{3e}-\frac{\sqrt{e}}{3}\left(\cos\frac{\sqrt{3}}{2}-\sqrt{3}\sin\frac{\sqrt{3}}{2}\right)$$

$$\sum_{k=0}^{\infty}\frac{x^{kn}}{(kn)!}=\frac{1}{n}\sum_{k=0}^{n-1}e^{\zeta_n^k x}$$

$$\sum_{k=0}^{\infty}\frac{(-1)^n x^{kn}}{(kn)!}=\frac{1}{n}\sum_{k=0}^{n-1}e^{\zeta_n^k e^{\pi i/n}x}$$

$$\sum_{k=0}^{\infty}\frac{x^{kn+m}}{(kn+m)!}=\frac{1}{n}\sum_{k=0}^{n-1}\zeta_n^{-km}e^{\zeta_n^k x}$$

$$\sum_{k=0}^{\infty}\frac{(-1)^k x^{kn+m}}{(kn+m)!}=\frac{e^{-m\pi i/n}}{n}\sum_{k=0}^{n-1}\zeta_n^{-km}e^{\zeta_n^k e^{\pi i/n}x}$$

$$\sum_{k=1}^{n}(-1)^{k+1} = \frac{1+(-1)^{n+1}}{2}$$
$$\sum_{k=1}^{n}(-1)^{k+1}k = \frac{1+(-1)^{n+1}(2n+1)}{4}$$
$$\sum_{k=1}^{n}(-1)^{k+1}k^2 = \frac{(-1)^{n+1}n(n+1)}{2}$$
$$\sum_{k=1}^{n}(-1)^{k+1}k^3 = \frac{(-1)^{n+1}(4n^3+6n^2-1)-1}{8}$$
$$\sum_{k=0}^{\infty} r^k = \frac{1}{1-r}$$
$$\sum_{k=0}^{\infty} kr^k = \frac{r}{(1-r)^2}$$
$$\sum_{k=0}^{\infty} k^2r^k = \frac{r(r+1)}{(1-r)^3}$$
$$\sum_{k=0}^{\infty} k^3r^k = \frac{r(r^2+4r+1)}{(1-r)^4}$$
$$\sum_{k=0}^{\infty} k^4r^k = \frac{r(r^3+11r^2+11r+1)}{(1-r)^5}$$
$$\sum_{k=0}^{\infty} k^nr^k = \frac{rA_n(r)}{(1-r)^{n+1}}$$
$$\sum_{k=0}^{\infty} \frac{k}{2^{k+1}} = 1$$
$$\sum_{k=0}^{\infty} \frac{k^2}{2^{k+1}} = 3$$
$$\sum_{k=0}^{\infty} \frac{k^3}{2^{k+1}} = 13$$
$$\sum_{k=0}^{\infty} \frac{k^4}{2^{k+1}} = 75$$
$$\sum_{k=0}^{\infty} \frac{k^n}{2^{k+1}} = \sum_{k=0}^{n} k!S_{n,k}$$
$$\sum_{k=0}^{\infty} \frac{k^n}{m^{k+1}} = \sum_{k=0}^{n} \frac{k!S_{n,k}}{(m-1)^{k+1}}$$
$$\sum_{k=0}^{\infty} k^nr^k = \frac{1}{r}\sum_{k=0}^{n} k!S_{n,k}\left(\frac{r}{1-r}\right)^{k+1}$$

$$\frac{1}{2^2-1}+\frac{1}{3^2-1}+\frac{1}{4^2-1}+\cdots=\frac{3}{4}$$

$$\frac{1}{3^2-2^2}+\frac{1}{4^2-2^2}+\frac{1}{5^2-2^2}+\cdots=\frac{25}{48}$$

$$\sum_{n=m+1}^{\infty}\frac{1}{n^2-m^2}=\frac{H_{2m}}{2m}$$

$$\frac{1}{1^3}-\frac{1}{3^3}+\frac{1}{5^3}-\frac{1}{7^3}+\cdots=\frac{\pi^3}{32}$$

$$\frac{1}{5^2}+\frac{1}{7^2}+\frac{1}{11^2}+\frac{1}{13^2}+\cdots=\frac{\pi^2}{9}-1$$

$$\frac{1}{1^2+1}+\frac{1}{2^2+1}+\frac{1}{3^2+1}+\cdots=\frac{1}{2}\left(\pi\frac{e^\pi+e^{-\pi}}{e^\pi-e^{-\pi}}-1\right)$$

$$\frac{1}{1^2+1}-\frac{1}{2^2+1}+\frac{1}{3^2+1}-\cdots=\frac{1}{2}-\frac{\pi}{e^\pi-e^{-\pi}}$$

$$\sum_{n=1}^{\infty}\frac{1}{n^2+a^2}=\frac{1}{2a}\left(\pi\coth a\pi-\frac{1}{a}\right)$$

$$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2+a^2}=\frac{1}{a}\left(\frac{1}{2a}-\frac{\pi}{e^{a\pi}-e^{-a\pi}}\right)$$

$$\sum_{k=0}^{n}r^k=\frac{1-r^{n+1}}{1-r}$$

$$\sum_{k=0}^{n}kr^k=\frac{r(1-(n+1)r^n+nr^{n+1})}{(1-r)^2}$$

$$\sum_{k=0}^{n}k^2r^k=\frac{r(r+1-(n+1)^2r^n-r^{n+1}-n^2r^{n+2})}{(1-r)^3}$$

$$\sqrt{1+\sqrt{1+\sqrt{1+\cdots}}}=\frac{1+\sqrt{5}}{2}$$

$$\sqrt{n+\sqrt{n+\sqrt{n+\cdots}}}=\frac{1+\sqrt{1+4n}}{2}$$

$$\sqrt[3]{6+\sqrt[3]{6+\sqrt[3]{6+\cdots}}}=2$$

$$\frac{1}{0!!}+\frac{1}{2!!}+\frac{1}{4!!}+\frac{1}{6!!}+\cdots=\sqrt{e}$$

$$\frac{1}{1!!}+\frac{1}{3!!}+\frac{1}{5!!}+\frac{1}{7!!}+\cdots=\sqrt{\frac{\pi e}{2}}\operatorname{erf}\frac{1}{\sqrt{2}}$$

$$\frac{1}{(0!)^2}+\frac{1}{(1!)^2}+\frac{1}{(2!)^2}+\frac{1}{(3!)^2}+\cdots=I_0(2)$$

$$\frac{1}{(0!)^2}-\frac{1}{(1!)^2}+\frac{1}{(2!)^2}-\frac{1}{(3!)^2}+\cdots=J_0(2)$$

$$\frac{1}{0!1!}+\frac{1}{1!2!}+\frac{1}{2!3!}+\frac{1}{3!4!}+\cdots=I_1(2)$$

$$\frac{1}{0!1!}-\frac{1}{1!2!}+\frac{1}{2!3!}-\frac{1}{3!4!}+\cdots=J_1(2)$$

$$\sum_{m=0}^{\infty}\frac{1}{m!(m+n)!}=I_n(2)$$

$$\sum_{m=0}^{\infty}\frac{(-1)^m}{m!(m+n)!}=J_n(2)$$

$$\sum_{n=1}^{\infty}\frac{x^n}{1-x^n}=\sum_{n=1}^{\infty}\sigma(n)x^n$$

$$\sum_{n=1}^{\infty}\frac{1}{x^n-1}=\sum_{n=1}^{\infty}\frac{\sigma(n)}{x^n}$$

$$B_n:=\sum_{k=0}^{n}S_{n,k}$$

$$\sum_{k=0}^{\infty}\frac{k^n}{k!}=eB_n$$

$$\sum_{k=0}^{\infty}\frac{k^n}{k!}x^k=e^x\sum_{k=0}^{n}S_{n,k}x^k$$

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}+\cdots=1$$

$$\frac{1}{1\cdot 2\cdot 3}+\frac{1}{2\cdot 3\cdot 4}+\frac{1}{3\cdot 4\cdot 5}+\cdots=\frac{1}{4}$$

$$\frac{1}{1\cdot 2\cdot 3\cdot 4}+\frac{1}{2\cdot 3\cdot 4\cdot 5}+\frac{1}{3\cdot 4\cdot 5\cdot 6}+\cdots=\frac{1}{18}$$

$$\frac{1}{1\cdot 2\cdot 3\cdot 4\cdot 5}+\frac{1}{2\cdot 3\cdot 4\cdot 5\cdot 6}+\cdots=\frac{1}{96}$$

$$\sum_{n=0}^{\infty}\frac{n!}{(n+m+1)!}=\frac{1}{m!m}$$

$$\sum_{n=0}^{\infty}\frac{1}{\binom{n+m}{n}}=\frac{m}{m-1}$$

$$\sum_{n=1}^{\infty}\frac{1}{n(n+m)}=\frac{H_m}{m}$$

$$\prod_{k=0}^{n}\frac{1}{x+k}=\sum_{k=0}^{n}\frac{(-1)^k\binom{n}{k}}{x+k}$$

$$\sum_{k=1}^{n}(-1)^{k+1}\binom{n}{k}H_k=\frac{1}{n}$$

$$\sum_{n=0}^{\infty}\frac{1}{(kn+m)(kn+m+k)}=\frac{1}{km}$$

$$\sqrt{5-\sqrt[3]{4}} = 1-\sqrt[3]{2}-\sqrt[3]{4}$$

$$\sqrt{4\sqrt[3]{2}-5} = \frac{1+2\sqrt[3]{2}-2\sqrt[3]{4}}{\sqrt{3}}$$

$$\sqrt{\sqrt[3]{44}+5} = \frac{1}{3}(1-\sqrt[3]{44}-\sqrt[3]{242})$$

$$\sqrt{4\sqrt[3]{10}+9} = \frac{1}{3}(1-2\sqrt[3]{10}-2\sqrt[3]{100})$$

$$\sqrt{1-4b+(2+b)\sqrt[3]{4b}} = 1+\sqrt[3]{4b}-\sqrt[3]{2b^2}$$

$$\frac{1}{1}-\frac{1}{4}+\frac{1}{7}-\cdots = \frac{1}{3}\left(\frac{\pi}{\sqrt{3}}+\ln 2\right)$$

$$\frac{1}{2}-\frac{1}{5}+\frac{1}{8}-\cdots = \frac{1}{3}\left(\frac{\pi}{\sqrt{3}}-\ln 2\right)$$

$$\int_0^1 \frac{x^{b-1}}{1-x^a}dx = \sum_{n=0}^{\infty}\frac{1}{an+b}$$

$$\int_0^1 \frac{x^{b-1}}{1+x^a}dx = \sum_{n=0}^{\infty}\frac{(-1)^n}{an+b}$$

$$\sum_{k=0}^{n}\frac{(-1)^k\binom{n}{k}}{2k+1} = \frac{(2n)!!}{(2n+1)!!}$$

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{n!(2n+1)}$$

$$\frac{\sqrt{\pi}}{2}e^{x^2}\operatorname{erf}(x) = \sum_{n=0}^{\infty}\frac{2^n x^{2n+1}}{(2n+1)!!}$$

$$\binom{n}{0}+\binom{n}{2}+\binom{n}{4}+\cdots = 2^{n-1}$$

$$\binom{n}{1}+\binom{n}{3}+\binom{n}{5}+\cdots = 2^{n-1}$$

$$\binom{n}{0}+\binom{n}{3}+\binom{n}{6}+\cdots = \frac{1}{3}\left(2^n+2\cos\frac{\pi n}{3}\right)$$

$$\binom{n}{1}+\binom{n}{4}+\binom{n}{7}+\cdots = \frac{1}{3}\left(2^n+2\cos\frac{\pi(n+4)}{3}\right)$$

$$\binom{n}{2}+\binom{n}{5}+\binom{n}{8}+\cdots = \frac{1}{3}\left(2^n+2\cos\frac{\pi(n+2)}{3}\right)$$

$$\sum_{k=0}^{\infty} \binom{n}{ak+b} = \frac{1}{a} \sum_{k=0}^{a-1} \zeta_a^{-kb} (1+\zeta_a^k)^n$$

$$f(x) = \sum_{n=0}^{\infty} c_n x^n$$

$$\sum_{n=0}^{\infty} c_{an+b} x^{an+b} = \frac{1}{a} \sum_{k=0}^{a-1} \zeta_a^{-kb} f(\zeta_a^k x)$$

$$\sum_{n=1}^{\infty} \frac{1}{4n^2-1} = \frac{1}{2}$$

$$\sum_{n=1}^{\infty} \frac{1}{9n^2-1} = \frac{1}{2} - \frac{\pi}{6\sqrt{3}}$$

$$\sum_{n=1}^{\infty} \frac{1}{16n^2-1} = \frac{1}{2} - \frac{\pi}{8}$$

$$\sum_{n=1}^{\infty} \frac{1}{36n^2-1} = \frac{1}{2} - \frac{\sqrt{3}\pi}{12}$$

$$\sum_{n=1}^{\infty} \frac{1}{64n^2-1} = \frac{1}{2} - \frac{1+\sqrt{2}}{16}\pi$$

$$\sum_{n=1}^{\infty} \frac{1}{m^2n^2-1} = \frac{1}{2} - \frac{\pi}{2m} \cot \frac{\pi}{m}$$

$$\pi \cot \pi z = \frac{1}{z} + 2 \sum_{n=1}^{\infty} \frac{z}{z^2-n^2}$$

$$\zeta(2n) = \frac{|B_{2n}|(2\pi)^{2n}}{2(2n)!}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} H_{2n}}{2n+1} = \frac{\pi \ln 2}{8}$$

$$\int_0^1 \frac{\ln(1+x)}{x} dx = \frac{\pi^2}{12}$$

$$\sum_{n=2}^{\infty} \frac{1}{(n^2-1)^2} = \frac{\pi^2}{12} - \frac{11}{16}$$

$$\sum_{n=2}^{\infty} \frac{n^2}{(n^2-1)^2} = \frac{\pi^2}{12} + \frac{1}{16}$$

$$\frac{d^n}{dx^n} \frac{\ln x}{x} = \frac{(-1)^{n+1} n!(H_n - \ln x)}{x^{n+1}}$$

$$\sum_{n=1}^{\infty} H_n x^n = -\frac{\ln(1-x)}{1-x}$$

$$\sum_{n=1}^{\infty} \frac{H_n}{2^{n+1}} = \ln 2$$

$$\int \frac{\ln x}{x^{n+1}} dx = -\frac{1 + n \ln x}{n^2 x^n} + C$$

$$\int \ln^n x \, dx = x \sum_{k=0}^{n} \frac{(-1)^k n!}{(n-k)!} \ln^{n-k} x + C$$

$$\frac{d^n}{dx^n} x^m e^x = e^x \sum_{k=0}^{n} \binom{n}{k} \frac{m!}{(m-k)!} x^{m-k}$$

$$\frac{d^n}{dx^n} e^{e^x} = e^{e^x} \sum_{k=0}^{n} S_{n,k} e^{kx}$$

$$e^{e^x} = e \sum_{n=0}^{\infty} \frac{B_n}{n!} x^n$$

$$B_n = \sum_{k=0}^{n} S_{n,k}$$

$$\frac{d^n}{dx^n} e^{ae^{bx}} = b^n e^{ae^{bx}} \sum_{k=0}^{n} S_{n,k} a^k e^{kx}$$

$$F_{0,k} = \delta_{0k}$$

$$F_{n+1,k} = F_{n,k-1} + 2kF_{n,k} - F_{n,k+1}$$

$$\frac{d^{2n}}{dx^{2n}} e^{\cosh x} = \frac{e^{\cosh x}}{2^{2n-1}} \left(\frac{F_{2n,0}}{2} + \sum_{k=1}^{2n} F_{2n,k} \cosh kx \right)$$

$$\frac{d^{2n+1}}{dx^{2n+1}} e^{\cosh x} = \frac{e^{\cosh x}}{2^{2n}} \sum_{k=1}^{2n+1} F_{2n+1,k} \sinh kx$$

$$\frac{d^{2n}}{dx^{2n}} e^{\cos x} = \frac{(-1)^n e^{\cos x}}{2^{2n-1}} \left(\frac{F_{2n,0}}{2} + \sum_{k=1}^{2n} F_{2n,k} \cos kx \right)$$

$$\frac{d^{2n+1}}{dx^{2n+1}} e^{\cos x} = \frac{(-1)^{n+1} e^{\cos x}}{2^{2n}} \sum_{k=1}^{2n+1} F_{2n+1,k} \sin kx$$

$$\prod_{n=1}^{\infty} \cos \frac{x}{2^n} = \frac{\sin x}{x}$$

$$\prod_{k=0}^{n-1} \cos 2^k x = \frac{\sin 2^n x}{2^n \sin x}$$

$$\cos^{2n+1} x = \frac{1}{2^{2n}} \sum_{k=0}^{n} \binom{2n+1}{n-k} \cos(2k+1)x$$

$$\cos^{2n} x = \frac{1}{2^{2n}} \left(\binom{2n}{n} + 2 \sum_{k=1}^{n} \binom{2n}{n-k} \cos 2kx \right)$$

$$\sin^{2n+1} x = \frac{1}{2^{2n}} \sum_{k=0}^{n} (-1)^k \binom{2n+1}{n-k} \sin(2k+1)$$

$$\sin^{2n} x = \frac{1}{2^{2n}} \left(\binom{2n}{n} + 2 \sum_{k=1}^{n} (-1)^k \binom{2n}{n-k} \sin 2kx \right)$$

$$\cos nx = \sum_{k=0}^{n} \binom{n}{k} \cos^{n-k} x \sin^k x \cos \frac{k\pi}{2}$$

$$\sin nx = \sum_{k=0}^{n} \binom{n}{k} \cos^{n-k} x \sin^k x \sin \frac{k\pi}{2}$$

$$x^{ex} \le e^{x^2}$$

$$\int_0^1 \ln x \ln(1+x)\,dx = 2 - 2\ln 2 - \frac{\pi^2}{12}$$

$$\int_0^1 \ln x \ln(1-x)\,dx = 2 - \frac{\pi^2}{6}$$

$$\frac{1}{x^n - 1} = \frac{1}{n}\sum_{k=0}^{n-1} \frac{\zeta_n^k}{x - \zeta_n^k}$$

$$\frac{1}{x^n + 1} = -\frac{1}{n}\sum_{k=0}^{n-1} \frac{\zeta_n^k}{e^{\pi i/n}x - \zeta_n^k}$$

$$\int x^n \ln^m x dx = x^{n+1}\sum_{k=0}^{m}(-1)^k \frac{m!\ln^{m-k} x}{k!(n+1)^{k+1}} + C$$

$$\int \frac{\ln^m x}{x^n} dx = -\frac{1}{x^{n-1}}\sum_{k=0}^{m} \frac{m!\ln^{m-k} x}{k!(n-1)^{k+1}} + C$$

$$\frac{d^n}{dx^n} f(g(x)) = n!\sum_{i=1}^{n} f^{(i)}(g(x)) \sum_{\substack{m_1+m_2+\cdots+m_n=i \\ m_1+2m_2+\cdots+nm_n=n}} \prod_{k=1}^{n} \frac{1}{m_k!}\left(\frac{g^{(k)}(x)}{k!}\right)^{m_k}$$

$$\prod_{n=1}^{\infty} \frac{(2k-1)(2k+1)}{(6k-1)(6k+1)} = \frac{2}{27}$$

$$\prod_{n=1}^{\infty}\left(1 - \frac{x^4}{n^4}\right) = \frac{\sin \pi x \sinh \pi x}{\pi^2 x}$$

$$\prod_{n=1}^{\infty}\left(1 + \frac{x^4}{n^4}\right) = \frac{\cosh 2\pi x - \cos 2\pi x}{4\pi^2 x^2}$$

$$\prod_{n=1}^{\infty}\left(1 + \frac{x^3}{n^3}\right) = \frac{1}{x^3\Gamma(x)\Gamma(\omega x)\Gamma(\omega^2 x)}$$

$$\prod_{n=1}^{\infty}\left(1 + \frac{1}{n^3}\right) = \frac{\cosh(\sqrt{3}\pi/2)}{\pi}$$

$$\prod_{n=1}^{\infty}\left(1 + \frac{2^3}{n^3}\right) = \frac{\sqrt{3}\sinh(\sqrt{3}\pi)}{2\pi}$$

$$\prod_{n=1}^{\infty}\left(1 + \frac{3^3}{n^3}\right) = \frac{7\cosh(3\sqrt{3}\pi/2)}{6\pi}$$

$$\prod_{n=1}^{\infty}\left(1 + \frac{4^3}{n^3}\right) = \frac{13\sqrt{3}\sinh(2\sqrt{3}\pi)}{12\pi}$$

$$\prod_{n=1}^{\infty}\left(1 + \frac{5^3}{n^3}\right) = \frac{133\cosh(5\sqrt{3}\pi/2)}{40\pi}$$

$$\prod_{n=1}^{\infty}\left(1 + \frac{(2m)^3}{n^3}\right) = \frac{m\sqrt{3}\sinh(m\sqrt{3}\pi)\prod_{k=0}^{m-1}(k^2+3m^2)}{(2m)!\pi}$$

$$\prod_{n=1}^{\infty}\left(1 + \frac{(2m+1)^3}{n^3}\right) = \frac{\cosh\{(2m+1)\sqrt{3}\pi/2\}\prod_{k=0}^{m-1}\{(2k+1)^2+3(2m+1)^2\}}{2^{2m}(2m+1)!\pi}$$

$$\prod_{n=1}^{\infty}\left(1-\frac{x^6}{n^6}\right)=\frac{\sin\pi x(\cosh\sqrt{3}\pi x-\cos\pi x)}{2\pi^3x^3}$$

$$\prod_{n=1}^{\infty}\left(1+\frac{x^6}{n^6}\right)=\frac{\sinh\pi x(\cosh\pi x-\cos\sqrt{3}\pi x)}{2\pi^3x^3}$$

$$\prod_{n=1}^{\infty}\left(1-\frac{1}{(2n)^6}\right)=\frac{4\cosh(\sqrt{3}\pi/2)}{\pi^3}$$

$$\prod_{n=1}^{\infty}\left(1-\frac{x^m}{n^m}\right)=\frac{1}{\prod_{k=0}^{m-1}\Gamma(1-\zeta_m^k x)}$$

$$\prod_{n=1}^{\infty}\left(1+\frac{x^{2m+1}}{n^{2m+1}}\right)=\frac{1}{x^{2m+1}\prod_{k=0}^{2m}\Gamma(\zeta_{2m+1}^k x)}$$

$$\prod_{n=1}^{\infty}\left(1-\frac{x^{4m+2}}{n^{4m+2}}\right)=\frac{\prod_{k=0}^{2m}\sin\zeta_{2m+1}^k\pi x}{(\pi x)^{2m+1}}$$

$$\prod_{n=1}^{\infty}\left(1-\frac{1}{(an+b)^2}\right)=\frac{\Gamma\left(\frac{b}{a}\right)^2}{\Gamma\left(\frac{b+1}{a}\right)\Gamma\left(\frac{b-1}{a}\right)}$$

$$\prod_{n=1}^{\infty}\left(1+\frac{1}{(an+b)^2}\right)=\frac{\Gamma\left(\frac{b}{a}\right)^2}{\Gamma\left(\frac{b+i}{a}\right)\Gamma\left(\frac{b-i}{a}\right)}$$

$$\prod_{n=1}^{\infty}\left(1+\frac{1}{an+b}\right)\left(1-\frac{1}{an+c}\right)=\frac{\Gamma\left(\frac{b}{a}\right)^2}{\Gamma\left(\frac{b+1}{a}\right)\Gamma\left(\frac{c-1}{a}\right)}$$

$$\prod_{k=m}^{n}(k+a)=\frac{\Gamma(n+a+1)}{\Gamma(m+a)}$$

$$\prod_{k=m}^{n}\left(1+\frac{a}{k}\right)=\frac{\Gamma(n+a+1)\Gamma(m)}{\Gamma(n+1)\Gamma(m+a)}$$

$$\sum_{i=1}^{n}a_i=0\Rightarrow\prod_{k=m}^{\infty}\prod_{i=1}^{n}\left(1+\frac{a_i}{k}\right)=\Gamma(m)^n\prod_{i=1}^{n}\frac{1}{\Gamma(m+a_i)}$$

$$\frac{\sin 2nx}{\sin x}=2\sum_{k=1}^{n}\cos(2k+1)x$$

$$\frac{\sin(2n+1)x}{\sin x}=1+2\sum_{k=1}^{n}\cos 2kx$$

$$\sin^2 x\cosh^2 x+\cos^2 x\sinh^2 x=\frac{\cosh 2x-\cos 2x}{2}$$

$$\prod_{n=0}^{\infty}\left(1-\frac{x^2}{(2n+1)^2}\right)=\cos\frac{\pi x}{2}$$
$$\prod_{n=0}^{\infty}\left(1+\frac{x^2}{(2n+1)^2}\right)=\cosh\frac{\pi x}{2}$$
$$\Gamma\left(\frac{1}{2}+z\right)\Gamma\left(\frac{1}{2}-z\right)=\frac{\pi}{\cos\pi z}$$
$$4\sin x\sin\left(\frac{\pi}{3}-x\right)\sin\left(\frac{\pi}{3}+x\right)=\sin 3x$$
$$4\cos x\cos\left(\frac{\pi}{3}-x\right)\cos\left(\frac{\pi}{3}+x\right)=\cos 3x$$
$$\tan x\tan\left(\frac{\pi}{3}-x\right)\tan\left(\frac{\pi}{3}+x\right)=\tan 3x$$
$$\sqrt[3]{\sqrt[3]{2}-1}=\frac{1-\sqrt[3]{2}+\sqrt[3]{4}}{\sqrt[3]{9}}$$
$$\sqrt[3]{5+4\sqrt[3]{2}+3\sqrt[3]{4}}=\frac{1+2\sqrt[3]{2}+\sqrt[3]{4}}{\sqrt[3]{9}}$$
$$\sqrt[3]{11+5\sqrt[3]{7}+3\sqrt[3]{49}}=\frac{1+\sqrt[3]{7}+\sqrt[3]{49}}{\sqrt[3]{9}}$$
$$\sqrt[3]{19+15\sqrt[3]{2}+12\sqrt[3]{4}}=1+\sqrt[3]{2}+\sqrt[3]{4}$$
$$\sqrt{\sqrt[3]{4}-1}=\frac{1-\sqrt[3]{2}+\sqrt[3]{4}}{\sqrt{3}}$$
$$\sqrt{3+2\sqrt[3]{2}+2\sqrt[3]{4}}=\frac{1+2\sqrt[3]{2}+\sqrt[3]{4}}{\sqrt{3}}$$
$$\sqrt{5+4\sqrt[3]{2}+3\sqrt[3]{4}}=1+\sqrt[3]{2}+\sqrt[3]{4}$$
$$\sqrt{5+3\sqrt[3]{7}+\sqrt[3]{49}}=\frac{1+\sqrt[3]{7}+\sqrt[3]{49}}{\sqrt{3}}$$

$$\prod_{k=0}^{n-1}\Gamma\left(z+\frac{k}{n}\right)=(2\pi)^{(n-1)/2}n^{1/2-nz}\Gamma(nz)$$

$$\sum_{k=0}^{n-1}\psi\left(z+\frac{k}{n}\right)=n\{\psi(nz)-\ln n\}$$

$$\sum_{k=0}^{n-1}\psi^{(m-1)}\left(z+\frac{k}{n}\right)=n^m\psi^{(m-1)}(nz)$$

$$\sum_{n=0}^{\infty}\frac{(-1)^n}{pn+q}=\frac{\pi}{2p\sin(q\pi/p)}+\frac{(-1)^{q+1}}{p}\sum_{k=1}^{[p/2]}\cos\frac{2qk\pi}{p}\ln\cos^2\frac{k\pi}{p}$$

$$\sum_{n=0}^{\infty}\frac{(-1)^n}{5n+1}=\frac{1}{5}(\sqrt{5}\ln(\sqrt{5}-1)+(1-\sqrt{5})\ln 2)+\frac{2\pi}{5\sqrt{10-2\sqrt{5}}}$$

$$\sum_{n=0}^{\infty}\frac{(-1)^n}{5n+2}=-\frac{1}{5}(\sqrt{5}\ln(\sqrt{5}-1)+(1-\sqrt{5})\ln 2)+\frac{2\pi}{5\sqrt{10+2\sqrt{5}}}$$

$$\sum_{n=0}^{\infty}\frac{(-1)^n}{5n+3}=\frac{1}{5}(\sqrt{5}\ln(\sqrt{5}+1)+(1-\sqrt{5})\ln 2)+\frac{2\pi}{5\sqrt{10+2\sqrt{5}}}$$

$$\sum_{n=0}^{\infty}\frac{(-1)^n}{5n+4}=-\frac{1}{5}(\sqrt{5}\ln(\sqrt{5}+1)+(1-\sqrt{5})\ln 2)+\frac{2\pi}{5\sqrt{10-2\sqrt{5}}}$$

$$\sum_{k=0}^{\infty}\frac{1}{(kn)!_n}=\sqrt[n]{e}$$

$$\sum_{k=0}^{\infty}\frac{1}{(mkn)!_n}=\frac{1}{m}\sum_{k=0}^{m-1}e^{\zeta_m^k/n}$$

$$\sum_{n=1}^{\infty}\frac{1}{(n+a)(n+b)}=\frac{\psi(b+1)-\psi(a+1)}{b-a}$$

$$\sum_{n=1}^{\infty}\frac{1}{n(2n+1)}=2-\ln 2$$

$$\sum_{n=1}^{\infty}\frac{1}{n(3n+1)}=3-\frac{3\ln 3}{2}-\frac{\pi}{2\sqrt{3}}$$

$$\sum_{n=1}^{\infty}\frac{1}{n(4n+1)}=4-\frac{\pi}{2}-3\ln 2$$

$$\sum_{n=1}^{\infty}\frac{1}{n(6n+1)}=6-\frac{\sqrt{3}\pi}{2}-\frac{3\ln 3}{2}-2\ln 2$$

$$\sum_{n=1}^{\infty}\frac{1}{n(pn+1)}=p-\ln(2p)-\frac{\pi}{2}\cot\frac{\pi}{p}+\sum_{k=1}^{n-1}\cos\frac{2\pi k}{p}\ln\sin\frac{\pi k}{p}$$

$$\sum_{n=1}^{\infty} k^n \sin nx = \frac{k \sin x}{1 - 2k \cos x + k^2}$$

$$\int_0^{\infty} \sin x^n dx = \sin \frac{\pi}{2n} \Gamma \left(1 + \frac{1}{n}\right)$$

$$\int_0^{\infty} \cos x^n dx = \cos \frac{\pi}{2n} \Gamma \left(1 + \frac{1}{n}\right)$$

$$x^n - x + a = 0$$

$$x = \sum_{k=0}^{\infty} \frac{\binom{nk}{k} a^{k(n-1)+1}}{k(n-1)+1}$$

$$\int e^{ae^{bx}} dx = \frac{a}{b} \mathrm{Ei}(ae^{bx}) + C$$

$$\int \frac{e^{x^n}}{x} dx = \frac{1}{n} \mathrm{Ei}(x^n) + C$$

$$\prod_{n=0}^{\infty} \left(1 + \frac{1}{2^{2^n}}\right) = 2$$

$$\prod_{n=0}^{\infty} \left(1 + \frac{1}{3^{2^n}}\right) = \frac{3}{2}$$

$$\prod_{n=0}^{\infty} \left(1 + x^{2^n}\right) = \frac{1}{1-x}$$

$$\prod_{n=0}^{\infty} \left(1 + x^{3^n} + x^{2 \cdot 3^n}\right)$$

$$\prod_{n=0}^{\infty} \sum_{k=0}^{m-1} x^{km^n} = \frac{1}{1-x}$$

$$\frac{1}{e}\sum_{n=1}^{\infty}\frac{H_n}{n!}=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n!n}$$

$$\mathrm{Ein}(z)=-\sum_{n=1}^{\infty}\frac{(-z)^n}{n!n}$$

$$\mathrm{Ein}(z)=e^{-z}\sum_{n=1}^{\infty}\frac{H_n}{n!}z^n$$

$$\sum_{n=0}^{\infty}\sum_{k=0}^{n}\frac{1}{2^{n+1}k!}=\sqrt{e}$$

$$\sum_{n=1}^{\infty}\frac{H_n}{2^n n}=\frac{\pi^2}{12}$$

$$\sum_{n=1}^{\infty}\frac{H_n^2}{n^2}=\frac{17\pi^4}{360}$$

$$\sum_{n=1}^{\infty}\frac{H_n}{n^3}=\frac{\pi^4}{72}$$

$$\sum_{n=1}^{\infty}\frac{H_n}{2^n n^2}=\zeta(3)-\frac{\pi^2\ln 2}{12}$$

$$H_n^{(m)}:=\sum_{k=1}^{n}\frac{1}{k^m}$$

$$\sum_{n=1}^{\infty}H_n^{(m)}z^n=\frac{\mathrm{Li}_m(z)}{1-z}$$

$$\sum_{n=1}^{\infty}\frac{H_n^{(q)}}{n^p}+\sum_{n=1}^{\infty}\frac{H_n^{(p)}}{n^q}=\zeta(p)\zeta(q)+\zeta(p+q)$$

$$\sum_{n=1}^{\infty}\frac{H_n^{(p)}}{n^p}=\frac{1}{2}\{\zeta(p)^2+\zeta(2p)\}$$

$$\sum_{n=1}^{\infty}\frac{H_n}{n^q}=\left(1+\frac{q}{2}\right)\zeta(q+1)-\frac{1}{2}\sum_{k=1}^{q-2}\zeta(k+1)\zeta(q-k)$$

$$\sum_{n=1}^{\infty}\frac{H_n}{n^q}=\frac{1}{2}\sum_{k=1}^{q-2}(-1)^{k+1}\zeta(k+1)\zeta(q-k)$$

$$q=2m+1$$

$$\frac{1}{e} = 1 - \cfrac{1}{2 - \cfrac{1}{3 - \cfrac{2}{4 - \cfrac{3}{5 - \cfrac{4}{\dots}}}}}$$

$$\sum_{n=0}^{\infty} \frac{1}{(n!)^m} = \cfrac{1}{1 - \cfrac{1}{2 - \cfrac{1}{2^m + 1 - \cfrac{2^m}{3^m + 1 - \cfrac{3^m}{\dots}}}}}$$

$$1 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{\dots}}}} = \frac{I_0(2)}{I_1(2)}$$

$$n + \cfrac{1}{n + 1 + \cfrac{1}{n + 2 + \cfrac{1}{n + 3 + \cfrac{1}{\dots}}}} = \frac{I_{n-1}(2)}{I_n(2)}$$

$$\frac{1}{\ln 2} = 1 + \cfrac{1^2}{1 + \cfrac{2^2}{1 + \cfrac{3^2}{1 + \cfrac{4^2}{\dots}}}}$$

$$\sum_{n=0}^{\infty} \left\{ \frac{(2n-1)!!}{(2n)!!} \right\}^2 \frac{1}{n+1} = \frac{4}{\pi}$$

$$\sum_{n=0}^{\infty} \frac{(3n)!}{3^{3n}(n+1)!(n!)^2} = \frac{9\sqrt{3}}{4\pi}$$

$$\prod_{k=1}^{n} k!k^k = (n!)^{n+1}$$

$$\sum_{n=0}^{\infty} \frac{(4k+1)!(k+1)}{(4k+6)!} = \frac{\pi}{96} - \frac{1}{32}$$

$$\int_0^1 \frac{2x \arctan x + \ln(1+x^2)}{1+x^2} dx = \frac{\pi \ln 2}{4}$$

$$\int_0^\infty e^{e^{-x^n}}\,dx = \frac{\Gamma(1/n)}{n}\sum_{k=1}^\infty \frac{1}{k!\sqrt[n]{k}}$$
$$\sum_{n=2}^\infty \frac{(-1)^n\zeta(n)}{n+1} = 1 + \frac{\gamma - \ln 2\pi}{2}$$
$$\int_0^z \ln\Gamma(z)\,dz = \frac{z(1-z)}{2} + \frac{z}{2}\ln 2\pi + z\ln\Gamma(z) - \ln G(1+z)$$
$$G(1+z) = \frac{z}{2}(\ln 2\pi - 1) - (1+\gamma)\frac{z^2}{2} + \sum_{n=2}^\infty \frac{(-1)^n\zeta(n)}{n+1}z^{n+1}$$
$$\int_1^\infty \frac{xe^{-x+1}-1}{x(x-1)} = -\gamma$$
$$\int_0^\infty \frac{e^{-ax}-e^{-bx}}{x}\,dx = \ln\frac{b}{a}$$
$$\int_0^\infty \left(\frac{1}{2} - \frac{1}{t} + \frac{1}{e^t-1}\right)\frac{e^{-t}}{t}\,dt = 1 - \frac{1}{2}\ln 2\pi$$
$$\lim_{n\to\infty} e^{-n}\sum_{k=0}^n \frac{n^k}{k!} = \frac{1}{2}$$
$$\frac{\zeta(2)}{2} - \frac{\zeta(2)}{3} + \frac{\zeta(4)}{4} - \frac{\zeta(4)}{5} + \cdots = \frac{1}{2}\ln\frac{2\pi}{e}$$
$$\ln\Gamma(z) = \frac{1}{2}\ln 2\pi + \left(z - \frac{1}{2}\right)\ln z - z + \int_0^\infty \left(\frac{1}{2} - \frac{1}{t} + \frac{1}{e^t-1}\right)\frac{e^{-zt}}{t}\,dt$$
$$\ln\Gamma(z) = \frac{1}{2}\ln 2\pi + \left(z - \frac{1}{2}\right)\ln z - z + 2\int_0^\infty \frac{\arctan\left(\frac{t}{z}\right)}{e^{2\pi t}-1}\,dt$$
$$\psi\left(\frac{p}{q}\right) = -\gamma - \ln 2q - \frac{\pi}{2}\cot\frac{p\pi}{q} + \sum_{k=1}^{q-1}\cos\frac{2pk\pi}{q}\ln\sin\frac{k\pi}{q}$$
$$\zeta(2n) = \frac{2}{2n+1}\sum_{k=1}^{n-1}\zeta(2k)\zeta(2n-2k)$$
$$\int_0^1 \frac{\ln(1+x^2)}{1+x^2}dx = \frac{\pi\ln 2}{2} - \beta(2)$$
$$\int_0^1 \frac{x\arctan x}{1+x^2}dx = \frac{\beta(2)}{2} - \frac{\pi\ln 2}{8}$$
$$\int_0^1 \frac{\ln^{2n}x}{1+x^2}dx = \frac{|E_{2n}|\pi^{2n+1}}{4^{n+1}}$$
$$\int_0^1 \frac{(-\ln x)^{s-1}}{1+x^2}dx = \Gamma(s)\beta(s)$$
$$\int_0^1 \frac{t}{e^t-1} = -\mathrm{Li}_2(1-e) - \frac{1}{2}$$

$$\sum_{n=0} \frac{1}{(an+b)^s} = \frac{1}{\Gamma(s)} \int_0^1 \frac{t^{s-1}e^{-bt}}{1-e^{-at}} dt$$

$$\sum_{n=0} \frac{(-1)^n}{(an+b)^s} = \frac{1}{\Gamma(s)} \int_0^1 \frac{t^{s-1}e^{-bt}}{1+e^{-at}} dt$$

$$\int_0^\infty e^{-x}(-\ln x)\,dx = \gamma$$

$$\int_0^\infty e^{-x}(-\ln x)^2\,dx = \frac{\pi^2}{6} + \gamma^2$$

$$\int_0^\infty e^{-x}(-\ln x)^3\,dx = 2\zeta(3) + \frac{\pi^2\gamma}{2} + \gamma^3$$

$$\int_0^\infty e^{-x}(-\ln x)^4\,dx = \frac{3\pi^4}{20} + 8\zeta(3)\gamma + \pi^2\gamma^2 + \gamma^4$$

$$\int_0^\infty e^{-x}(-\ln x)^n\,dx = n! \sum_{m_1+2m_2+\cdots+nm_n=n} \frac{\gamma^{m_1}}{m_1!} \prod_{k=1}^{n} \frac{1}{m_k!} \left\{\frac{\zeta(k)}{k}\right\}^{m_k}$$

$$\sum_{n=0}^{\infty} \frac{(2n-1)!!}{(2n)!!(2n+1)^2} = \frac{\pi \ln 2}{2}$$

$$\int_0^\infty x^{s-1} \sin x\,dx = \Gamma(s) \sin \frac{\pi s}{2}$$

$$\int_0^\infty x^{s-1} \cos x\,dx = \Gamma(s) \cos \frac{\pi s}{2}$$

$$\int_0^\infty \frac{(\sin x - \cos x)\ln x}{\sqrt{x}} dx = \frac{\pi^{3/2}}{\sqrt{2}}$$

$$\int_0^1 \frac{\arcsin x}{x} dx = \int_0^{\pi/2} x \cot x \, dx = \frac{\pi \ln 2}{2}$$

$$\int_0^1 \ln\ln x \, dx = -\gamma + \pi i$$

$$\int_0^{\pi/2} \frac{\tan x}{e^{\tan x} - 1} dx = -\frac{1}{2}\left\{\ln 2\pi + \pi + \psi\left(\frac{1}{2\pi}\right)\right\}$$

$$\int_0^\infty \frac{tdt}{(1+t^2)(e^{2\pi t}-1)} = \frac{\gamma}{2} - \frac{1}{4}$$

$$\int_0^\infty \frac{tdt}{(1+t^2)^2(e^{2\pi t}-1)} = \frac{\pi^2}{24} - \frac{3}{8}$$

$$\sum_{n=-\infty}^\infty e^{-\pi n^2 x} = \frac{1}{\sqrt{x}} \sum_{n=-\infty}^\infty e^{-\pi n^2/x}$$

$$\sum_{n=1}^\infty \frac{n^5}{e^{2\pi n} - 1} = \frac{1}{504}$$

$$\sum_{n=1}^\infty \frac{n^9}{e^{2\pi n} - 1} = \frac{1}{264}$$

$$\sum_{n=1}^\infty \frac{n^{13}}{e^{2\pi n} - 1} = \frac{1}{24}$$

$$\sum_{n=1}^\infty \frac{n \coth \pi n}{n^4 - x^4} = \frac{1}{4\pi x^4} - \frac{\pi}{4x^2} \cot \pi x \coth \pi x$$

$$\sum_{n=1}^\infty \frac{\coth \pi n}{n^3} = \frac{7\pi^3}{180}$$

$$\sum_{n=1}^\infty \frac{\coth \pi n}{n^7} = \frac{19\pi^7}{56700}$$

$$\int_{-\infty}^\infty \frac{\cos xy}{x^2 + a^2} dx = \frac{\pi e^{-ay}}{a}$$

$$\int_0^{\pi/2} \ln^2 \sin x \, dx = \frac{\pi \ln^2 2}{2} + \frac{\pi^3}{24}$$

$$\int_0^{\pi/2} x \ln \sin x \, dx = -\frac{\pi^2 \ln 2}{8} - \frac{7\zeta(3)}{16}$$

$$\int_0^{\pi/2} x \ln \sin x \, dx = -\frac{\pi^2 \ln 2}{8} + \frac{7\zeta(3)}{16}$$

$$\int_0^{\pi/2} x^2 \ln \sin x \, dx = -\frac{\pi\zeta(3)}{4} - \frac{\pi^3 \ln 2}{24}$$

$$\int_0^{\pi/2} x^2 \ln \sin x \, dx = \frac{3\pi\zeta(3)}{16} - \frac{\pi^3 \ln 2}{24}$$

$$\int_0^{\pi/2} \ln^3 \sin x \, dx = -\frac{3\pi\zeta(3)}{4} - \frac{\pi^3 \ln 2}{8} - \frac{\pi \ln^3 2}{2}$$

$$\int_0^{\pi/2} \ln \sin x \ln \cos x \, dx = \frac{\pi \ln^2 2}{2} - \frac{\pi^3}{48}$$

$$\int_0^{\infty} \frac{\arctan x}{x^{\ln x + 1}} dx = \frac{\pi^{3/2}}{4}$$

$$\int_0^{\pi/2} \sin \sin x \, dx = 1 - \frac{1}{3!!^2} + \frac{1}{5!!^2} - \frac{1}{7!!^2} + \cdots$$

$$\int_0^{\pi/2} \cos \cos x \, dx = \frac{\pi J_0(1)}{2}$$

$$\sum_{n=1}^{\infty} \frac{(2n-2)!}{2^{2n} n!^2} = 1 - \ln 2$$

$$\sum_{n=0}^{\infty} \frac{(2n)!!}{(2n+1)!!(2n+1)} = 2G$$

$$\int_0^{\pi/4} x^n \ln \tan x \, dx = n! \sum_{k=0}^{[n/2]} \frac{(-1)^{k+1} 4^{k-n} \pi^{n-2k} \beta(2k+2)}{(n-2k)!} + \frac{n! \sin(n\pi/2)}{2^n} \left(1 - \frac{1}{2^{n+2}}\right) \zeta(n+2)$$

$$\int_0^{\pi/2} x^n \ln \left(2 \sin \frac{x}{2}\right) \, dx = n! \sum_{k=0}^{[n/2]} \frac{(-1)^{k+1} \beta(2k+2)}{(n-2k)!} \left(\frac{\pi}{2}\right)^{n-2k}$$

$$+ \frac{n!}{2^{n+2}} \sum_{k=1}^{[(n-1)/2]} \frac{(-1)^{k+1} \pi^{n-2k+1}}{(n-2k)!} \left(1 - \frac{1}{2^{2k}}\right) \zeta(2k+1) + n! \cos \frac{\pi n}{2} \zeta(n+2)$$

$$\int_0^{\pi/2} \ln^4 \sin x \, dx = \frac{19\pi^5}{480} + \frac{\pi^3 \ln^2 2}{4} + \frac{\pi \ln^4 2}{2} + 3\pi \ln 2\zeta(3)$$

$$\int_0^{\pi/2} x^2 \ln^2 \sin x \, dx = \frac{11\pi^5}{1440} + \frac{\pi^3 \ln^2 2}{24} + \frac{\pi\zeta(3) \ln 2}{2}$$

$$\int_0^{\pi/2} x \ln^2 \sin x \, dx = \frac{79\pi^4}{2880} + \frac{\pi^2 \ln^2 2}{6} - \frac{\ln^4 2}{24} - \mathrm{Li}_4\left(\frac{1}{2}\right)$$

$$\int_0^{\pi/2} x \ln^2 \sin x \, dx = -\frac{19\pi^4}{2880} + \frac{\pi^2 \ln^2 2}{12} + \frac{\ln^4 2}{24} + \mathrm{Li}_4\left(\frac{1}{2}\right)$$

$$\int_0^{\pi/2} x \ln \sin x \ln \cos x \, dx = \frac{\pi^2 \ln^2 2}{8} - \frac{\pi^4}{192}$$

$$\int_0^{\pi/2} \sin x \ln \cos x \, dx = -1$$

$$\int_0^{\pi/2} \sin x \ln \cos x \, dx = \ln 2 - 1$$

$$\int_0^{\pi/2} x \sin x \ln \cos x \, dx = \ln 2 - 2$$

$$\int_0^{\pi/2} x \cos x \ln \sin x \, dx = 2 - \ln 2 - \frac{\pi}{2}$$

$$\int_0^{\pi/2} \cos ax \ln \sin ax \, dx = \frac{\sin \frac{\pi a}{2}}{a}\left(\ln \sin \frac{\pi a}{2} - 1\right)$$

$$\int_0^{\pi/2} \sin ax \ln \cos ax \, dx = \frac{\cos \frac{\pi a}{2}}{a}\left(1 - \ln \cos \frac{\pi a}{2}\right)$$

$$\int_0^{\pi/2} \sin x \ln \sin x \, dx = 2(\beta(2) - 1)$$

$$\int_0^{\pi/2} \cos x \ln \cos x \, dx = \frac{\pi}{2}(\ln 2 - 1) + 2(1 - \beta(2))$$

$$\sum_{n=0}^{\infty} \frac{(2n)!!}{(2n+1)!!(2n+3)^2} = 2(1 - \beta(2))$$

$$\sum_{n=0}^{\infty} \frac{(2n)!!}{(2n+1)!!(2n+1)} = 2\beta(2)$$

$$\sum_{n=0}^{\infty} \frac{(2n)!!(n+1)(2n+5)}{(2n+1)!!(2n+1)(2n+3)^2} = 1$$

$$1+\frac{1}{2}\frac{1}{3}+\frac{1\cdot 3}{2\cdot 4}\frac{1}{5}+\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{1}{7}+\cdots=\frac{\pi}{2}$$

$$1+\frac{1}{2}\frac{1}{3^2}+\frac{1\cdot 3}{2\cdot 4}\frac{1}{5^2}+\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{1}{7^2}+\cdots=\frac{\pi\ln 2}{2}$$

$$1+\frac{1}{2}\frac{1}{3^3}+\frac{1\cdot 3}{2\cdot 4}\frac{1}{5^3}+\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{1}{7^3}+\cdots=\frac{12\pi\ln^2 2+\pi^3}{48}$$

$$1+\frac{1}{2}\frac{1}{3^4}+\frac{1\cdot 3}{2\cdot 4}\frac{1}{5^4}+\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{1}{7^4}+\cdots=\frac{6\pi\zeta(3)+\pi^3\ln 2+4\pi\ln^3 2}{48}$$

$$\sum_{n=-\infty}^{\infty}\frac{\pi}{\cosh\pi^2 n}=\sum_{n=\infty}^{\infty}\frac{1}{\cosh n}$$

$$\sin(x\sin\theta)=\sum_{n=-\infty}^{\infty}J_n(x)\sin(n\theta)$$

$$e^{ix\sin\theta}=\sum_{n=-\infty}^{\infty}J_n(x)e^{in\theta}$$

$$\int_0^{\pi/2} \sin ax \cos^a x\, dx = \frac{1}{2^{a+1}} \int_{-1}^{1} \frac{(x+1)^a - 1}{x}\, dx = \frac{1}{2^a} \sum_{n=0}^{\infty} \frac{\binom{a}{2n+1}}{2n+1}$$

$$\int_0^{\pi/2} \sin nx \cos^n x\, dx = \frac{1}{2^{n+1}} \sum_{k=1}^{n} \frac{2^k}{k}$$

$$\int_0^{\pi/2} \cos bx \cos^{a-1} x\, dx = \frac{\pi}{2^a} \frac{\Gamma(a)}{\Gamma\left(\frac{1+a+b}{2}\right)\Gamma\left(\frac{1+a-b}{2}\right)}$$

$$\int_0^1 \frac{\ln^2(1+x)}{x}\, dx = \frac{\zeta(3)}{4}$$

$$\int_0^a \frac{\ln^n(1+x)}{x}\, dx = \frac{\ln^{n+1}(1+a)}{n+1} + n!\zeta(n+1) - \sum_{k=0}^{n} \frac{n!}{(n-k)!} \mathrm{Li}_{k+1}\left(\frac{1}{1+a}\right) \ln^{n-k}(1+a)$$

$$\int_0^1 \frac{\ln^3(1+x)}{x}\, dx = \frac{\pi^4}{15} + \frac{\pi^2 \ln^2 2}{4} - \frac{\ln^4}{4} - \frac{21\zeta(3)\ln 2}{4} - 6\mathrm{Li}_4\left(\frac{1}{2}\right)$$

$$\int_0^{\pi/2} \frac{x}{\sin x} dx = 2\beta(2)$$

$$\int_0^{\pi/2} \frac{x^2}{\sin^2 x} dx = \pi \ln 2$$

$$\int_0^{\pi/2} \frac{x^3}{\sin^3 x} dx = -\frac{3\pi^2}{8} + \left(6 + \frac{3\pi^2}{4}\right)\beta(2) - 6\beta(4)$$

$$\int_0^{\infty} [e^x] e^{-sx}\, dx = \int_1^{\infty} \frac{[x]}{x^{s-1}} dx = \frac{\zeta(s)}{s}$$

$$\int_0^1 \frac{\ln(1-x)\ln x}{x} dx = \zeta(3)$$

$$\int_0^1 \frac{\ln^2(1-x)\ln x}{x} dx = -\frac{\pi^4}{180}$$

$$\int_0^1 \frac{\ln^3(1-x)\ln x}{x} dx = 12\zeta(5) - \pi^2\zeta(3)$$

$$\frac{1}{2} + \frac{2}{1\cdot 3}\frac{1}{4} + \frac{2\cdot 4}{1\cdot 3\cdot 5}\frac{1}{6} + \cdots = \frac{\pi^2}{8}$$

$$\frac{1}{2^2} + \frac{2}{1\cdot 3}\frac{1}{4^2} + \frac{2\cdot 4}{1\cdot 3\cdot 5}\frac{1}{6^2} + \cdots = \frac{\pi^2 \ln 2}{8} - \frac{7\zeta(3)}{16}$$

$$\int_0^{\infty} (-\ln\tan x)^{s-1}\, dx = \Gamma(s)\beta(s)$$

$$-\int_0^{\infty} \ln(-\ln\tan x)\, dx = \frac{\gamma\pi}{4} + \frac{\ln 1}{1} - \frac{\ln 3}{3} + \frac{\ln 5}{5} - \frac{\ln 7}{7} + \cdots$$

$$-\int_0^{\infty} \frac{\ln x}{e^x + e^{-x}} dx = \frac{\gamma\pi}{4} + \frac{\ln 1}{1} - \frac{\ln 3}{3} + \frac{\ln 5}{5} - \frac{\ln 7}{7} + \cdots$$

$$\int_0^{\infty} \frac{x\sin x}{1+x^2} dx = \frac{\pi}{2e}$$

$$\int_0^{\infty} \sin x \arctan\frac{1}{x}\, dx = \frac{\pi}{2} - \frac{\pi}{2e}$$

$$\int_0^{\pi/2} \sqrt{\tan x}\, dx = \frac{\pi}{\sqrt{2}}$$

$$\int_0^{\pi/2} \frac{1}{\sqrt{\tan x}} dx = \frac{\pi}{\sqrt{2}}$$

$$\int_0^{\pi/2} \sqrt[3]{\tan x}\, dx = \frac{\pi}{\sqrt{3}}$$

$$\int_0^{\infty} \frac{f(ax) - f(bx)}{x} dx = \{f(\infty) - f(0)\} \ln \frac{a}{b}$$

$$\int_0^{\infty} \cos(\tan x - \cot x)\, dx = \frac{\pi}{e^2}$$

$$\int_0^{\infty} \frac{(1 - \cos ax) \ln x}{x^2} dx = \frac{\pi a(\gamma - 1 - \ln a)}{2}$$

$$\int_0^{\infty} \frac{(\cos^2 x - \cos x) \ln x}{x^2} dx = \frac{\pi \ln 2}{2}$$

$$\int_0^{\pi/2} x^2 \sin x \ln \sin x \, dx = 6 - \frac{\pi^2}{4} - 2\ln 2 + \frac{\pi^2 \ln 2}{4} - \frac{7\zeta(3)}{2}$$

$$\int_0^{\pi/2} x^2 \sin^2 x \ln \sin x \, dx = \frac{\pi}{16} + \frac{\pi^3}{6} + \frac{\pi \ln 2}{8} - \frac{\pi^3 \ln 2}{48} - \frac{\pi\zeta(3)}{8}$$

$$\int_0^{\pi/2} x^2 \sqrt{\sin x} \, dx = \frac{\pi\sqrt{2\pi}\psi'\left(\frac{5}{4}\right)}{\Gamma\left(\frac{1}{4}\right)^2}$$

$$\int_0^{\pi/2} \ln^2(2\sin x) \, dx = \frac{\pi^3}{24}$$

$$\int_0^{\pi/2} \ln^3(2\sin x) \, dx = -\frac{3\zeta(3)\pi}{4}$$

$$\int_0^{\pi/2} \ln^4(2\sin x) \, dx = \frac{19\pi^5}{480}$$

$$\int_0^{\pi/2} \ln^5(2\sin x) \, dx = -\frac{5\pi^3\zeta(3)}{8} - \frac{45\pi\zeta(5)}{4}$$

$$\int_0^{\pi/2} \ln^6(2\sin x) \, dx = \frac{275\pi^7}{2688} + \frac{45\pi\zeta(3)^2}{4}$$

$$\int_0^{\pi/2} x^2 \ln(2\sin x) \, dx = -\frac{\pi\zeta(3)}{4}$$

$$\int_0^{\pi/2} x^2 \ln^2(2\sin x) \, dx = \frac{11\pi^5}{1440}$$

$$\int_0^{\pi/2} x^2 \ln^3(2\sin x) \, dx = -\frac{\pi^3\zeta(3)}{8} - \frac{3\pi\zeta(5)}{4}$$

$$\int_0^{\pi/2} x^2 \ln^4(2\sin x) \, dx = \frac{33\pi^7}{4480} + \frac{3\pi\zeta(3)^2}{2}$$

$$\int_0^{\pi/2} x^4 \ln(2\sin x) \, dx = -\frac{\pi^3\zeta(3)}{8} + \frac{3\pi\zeta(5)}{4}$$

$$\int_0^{\pi/2} x^4 \ln^2(2\sin x) \, dx = \frac{5\pi^7}{8064} + \frac{3\pi\zeta(3)^2}{4}$$

$$\int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$$

$$\int_0^{\infty} \frac{\sin^2 x}{x^2} dx = \frac{\pi}{2}$$

$$\int_0^{\infty} \frac{\sin^3 x}{x^3} dx = \frac{3\pi}{8}$$

$$\int_0^{\infty} \frac{\sin^4 x}{x^4} dx = \frac{\pi}{3}$$

$$\int_0^{\infty} \frac{\sin^n x}{x^n} dx = \frac{\pi}{2^n (n-1)!} \sum_{k=0}^{[n/2]} (-1)^k \binom{n}{k} (n-2k)^{n-1}$$

$$\int_0^\infty \frac{\sin^{2n+1} x}{x} dx = \frac{\pi}{2}\binom{2n}{n}$$

$$\lim_{n\to\infty} \sqrt{n} \int_0^\infty \left(\frac{2\sin x}{ex}\right)^n dx = \frac{e}{4}\sqrt{\frac{\pi}{2}}$$

$$\int_0^\infty \frac{\sin x}{x(1+\cos^2 x)} dx = \frac{\pi}{2\sqrt{2}}$$

$$\int_0^\infty \frac{\sin x}{x(a+\cos^2 x)} dx = \frac{\pi}{2\sqrt{a(a+1)}}$$

$$\int_0^\infty \frac{\sin x \ln(1+\sec^2 x)}{x} dx = \pi \ln(1+\sqrt{2})$$

$$\int_0^\infty \frac{\sin x \ln(a+\sec^2 x)}{x} dx = \pi \ln(a+\sqrt{a^2+1})$$

$$\int_0^\infty \frac{\sin x}{x} \ln\left(\frac{a+\cos^2 x}{b+\cos^2 x}\right) dx = \pi \ln\left(\frac{a+\sqrt{a^2+1}}{b+\sqrt{b^2+1}}\right)$$

$$\sum_{n=2}^\infty \frac{(-1)^n \zeta(n)}{n} = \gamma$$

$$\int_0^1 \frac{1}{1-x} + \frac{1}{\ln x} dx = \gamma$$

$$\frac{\arcsin x}{\sqrt{1-x^2}} = \sum_{n=0}^{\infty} \frac{(2n)!!}{(2n+1)!!} x^{2n+1} = \sum_{n=0}^{\infty} \frac{2^{2n}}{\binom{2n}{n}} \frac{x^{2n+1}}{2n+1}$$

$$\int_0^1 x^{s-tx}\,dx = \sum_{n=1}^{\infty} \frac{t^{n-1}}{(n+s)^n}$$

$$\int_0^1 \frac{dx}{1+x\ln x} = \sum_{n=1}^{\infty} \frac{(n-1)!}{n^n}$$

$$\int_0^1 \frac{dx}{(1+x\ln x)^2} = \sum_{n=1}^{\infty} \frac{n!}{n^n}$$

$$\int_0^1 (-x\ln x)^{s-1}\,dx = \frac{\Gamma(s)}{s^s}$$

$$\int_0^{\infty} \operatorname{sech}^s x\,dx = \frac{\sqrt{\pi}\Gamma\left(\frac{s}{2}\right)}{2\Gamma\left(\frac{s+1}{2}\right)}$$

$$\int_0^x \tan^n x\,dx = (-1)^{m+1} x + \sum_{k=1}^{m} \frac{(-1)^{k+1}\tan^{n-2k+1} x}{n-2k+1}$$

$n = 2m$

$$\int_0^x \tan^n x\,dx = (-1)^{m+1} \ln|\cos x| + \sum_{k=1}^{m} \frac{(-1)^{k+1}\tan^{n-2k+1} x}{n-2k+1}$$

$n = 2m+1$

$$\int_0^x \frac{dx}{\sin^n x} = \frac{1}{2^{n-1}} \sum_{k=0}^{n-1} \frac{\binom{n-1}{k}\tan^{2k-n+1}\left(\frac{x}{2}\right)}{2k-n+1}$$

$n = 2m$

$$\int_0^x \frac{dx}{\sin^n x} = \frac{1}{2^{n-1}} \left\{ \binom{n-1}{m} \ln\tan\left(\frac{x}{2}\right) + \sum_{k=0, k\neq m}^{n-1} \frac{\binom{n-1}{k}\tan^{2k-n+1}\left(\frac{x}{2}\right)}{2k-n+1} \right\}$$

$n = 2m+1$

$$\int_0^{\pi/2} \frac{\arctan\sin x}{\sin x} dx = \frac{\pi\ln(1+\sqrt{2})}{2}$$

$$\int_0^{\pi/2} \frac{\arctan(t\sin x)}{\sin x} dx = \frac{\pi\ln(t+\sqrt{t^2+1})}{2}$$

$$\int_0^a \arcsin \operatorname{sech} x\,dx = \int_0^{\pi/2} \ln(\cosh a - \sin x)\,dx + 2\beta(2) + \frac{\pi\ln 2}{2}$$

$$\prod_{k=m}^{n} \frac{1}{x+a_k} = \sum_{k=m}^{n} \frac{1}{x+a_k} \prod_{i\neq k} \frac{1}{a_i - a_k}$$

$$\prod_{k=1}^{n} \frac{1}{kx+1} = \frac{1}{n!} \sum_{k=1}^{n} \frac{(-1)^{n-k}\binom{n}{k} k^n}{kx+1}$$

$$n! = \sum_{k=0}^{n} (-1)^{n-k} \binom{n}{k} k^{n-1}$$

$$\left(\prod_{k=m}^{n} a_k\right)^{-1} = \sum_{k=m}^{n} \left\{ a_k \prod_{i\neq k} (a_i - a_k) \right\}^{-1}$$

$$\prod_{k=0}^{n} \frac{1}{x+k^2} = 2\sum_{k=0}^{n} \frac{(-1)^k}{(x+k^2)(n-k)!(n+k)!}$$

$$\prod_{k=1}^{n} \frac{1}{k^2x+1} = 2\sum_{k=1}^{n} \frac{(-1)^{n-k}k^{2n}}{(k^2x+1)(n-k)!(n+k)!}$$

$$\sum_{k=0}^{n} (-1)^{n-k} \binom{2n}{n-k} k^{2n} = \frac{(2n)!}{2}$$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{(-1)^{n-k}k^{2n}(n!)^2}{(k^2+1)(n-k)!(n+k)!} = \frac{\pi}{e^{\pi}-e^{-\pi}}$$

$$\sum_{n=1}^{p}\frac{(-1)^n}{n}\left(\sum_{k_1+k_2+\cdots+k_n=p}\prod_{i=1}^{n}\frac{1}{k_i!}\right)=0$$

$p\geq 2$

$$\sum_{n=1}^{p}\frac{(-1)^n}{n!}\left(\sum_{k_1+k_2+\cdots+k_n=p}\prod_{i=1}^{n}\frac{1}{k_i}\right)=0$$

$p\geq 2$

$$\left(\sum_{i=1}^{m}x_i\right)^n=n!\sum_{k_1+k_2+\cdots+k_m=n}\prod_{i=1}^{m}\frac{x_i^{k_i}}{k_i!}$$

$$\prod_{i=1}^{n}\sum_{j=1}^{m}x_{ij}=\sum_{\phi\in M}\prod_{i=1}^{n}x_{i\phi(i)}$$

$M=\mathrm{Map}(\{1,2,\cdots,n\},\{1,2,\cdots,m\})$

$$\int_0^1\int_0^1\frac{x^{a-1}y^{b-1}}{\sqrt{-\ln(xy)}}dxdy=\frac{\sqrt{\pi}}{a\sqrt{b}+b\sqrt{a}}$$

$$\int_0^1\cdots\int_0^1\frac{\prod_{k=1}^{n}x_k^k\,dx_k}{\sqrt{-\ln\prod_{k=1}^{n}x_k}}=\frac{\sqrt{\pi}}{n!}\sum_{k=1}^{n}(-1)^{k-1}\binom{n}{k}\sqrt{k}$$

$$\sum_{k=1}^{\infty}\sum_{n=0}^{\infty}\frac{1}{kH_{n+k}(n+k)^s}=\zeta(s)$$

$$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{(-1)^{n+m}}{n^2+m^2}=\frac{\pi^2}{24}-\pi\ln\prod_{n=0}^{\infty}(1+e^{-(2n+1)\pi})$$

$$\sum_{n=-\infty}^{\infty}\frac{1}{(x+n)^2}=\frac{\pi^2}{\sin^2\pi x}$$

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^3\sinh n\pi}=\frac{\pi^3}{360}$$

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^7\sinh n\pi}=\frac{13\pi^7}{453600}$$

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^{11}\sinh n\pi}=\frac{4009\pi^{11}}{13621608000}$$

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^{4m-1}\sinh n\pi}=\frac{\pi^{4m-1}}{(4m)!}\sum_{k=0}^{2m}(-1)^{2m-k}\binom{4m}{2k}D_kD_{2m-k}$$

$D_k=B_{2k}(2^{2k-1}-1)$

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}n}{\sinh n\pi}=\frac{1}{4\pi}$$

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}n^5}{\sinh n\pi}=0$$

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}n^{4m+1}}{\sinh n\pi}=0$$

$$\sum_{n=0}^{\infty} \mathrm{Li}_3(-e^{-(2n+1)\pi}) = -\frac{\pi^3}{720}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\left(n+\frac{1}{2}\right)\cosh\left(n+\frac{1}{2}\right)\pi} = \frac{\pi}{4}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\left(n+\frac{1}{2}\right)^5\cosh\left(n+\frac{1}{2}\right)\pi} = \frac{\pi^5}{24}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\left(n+\frac{1}{2}\right)^9\cosh\left(n+\frac{1}{2}\right)\pi} = \frac{23\pi^9}{3360}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{\left(n+\frac{1}{2}\right)^{4m+1}\cosh\left(n+\frac{1}{2}\right)\pi} = \frac{\pi^{4m+1}}{4(4m)!}\sum_{k=0}^{2m}(-1)^k\binom{4m}{2k}E_{2k}E_{4m-2k}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n\left(n+\frac{1}{2}\right)^{4m-1}}{\cosh\left(n+\frac{1}{2}\right)\pi} = 0$$